

Visokošolske enačbe

Kompleksna števila - Vektorji in matrike - Večkratne funkcije - Krivulje in ploskve - Prostorska polja - Statistika - Dinamika - Relativnost - Termodinamika - Statična E & M polja - Elektromagnetni valovi - Elektroni in ioni - Kvantni delci - Valovna mehanika - Molekule, kristali, plini - Atomska jedra - Zvezde in vesolje

Kompleksna števila

Fazorji	$\hat{u} = (u_1, u_2) = (u \cos \varphi, u \sin \varphi)$ $u = \sqrt{(u_1^2 + u_2^2)} = \hat{u} $ $\varphi = \operatorname{atan} \frac{u_2}{u_1} = \operatorname{Arg}(\hat{u})$ $\operatorname{Re}(\hat{u}) = u_1$ $\operatorname{Im}(\hat{u}) = u_2$
Računska pravila	$c\hat{u} = (cu_1, cu_2)$ $\hat{u} + \hat{v} = (u_1 + v_1, u_2 + v_2)$ $\hat{u} \hat{v} = uv (\cos(\alpha + \beta), \sin(\alpha + \beta))$ $\hat{u} / \hat{v} = (u/v) (\cos(\alpha - \beta), \sin(\alpha - \beta))$ $\hat{u} \hat{v} = (u_1 v_1 - u_2 v_2, u_1 v_2 + u_2 v_1)$ $\hat{u} / \hat{v} = (u_1 v_1 + u_2 v_2, u_2 v_1 - u_1 v_2)$
Imaginarna enota	$\hat{u} = u_1 \cdot (1, 0) + u_2 \cdot (0, 1)$ $\hat{u} = u_1 + iu_2$ $i = (0, 1)$ $i \cdot i = i^2 = -1$
Konjugirani fazorji	$\hat{u}^* = u_1 - iu_2$ $ \hat{u} ^2 = \hat{u} \hat{u}^*$
Potenca in eksponencial	$\hat{u}^n = \hat{u} \cdot \hat{u} \dots \hat{u}$ $\hat{u}^n = u^n (\cos n\varphi + i \sin n\varphi)$ $\hat{u}^s = u^s (\cos s\varphi + i \sin s\varphi)$ $e^{i\varphi} = \cos \varphi + i \sin \varphi$ $e^{\hat{u}} = e^{u_1 + iu_2} = e^{u_1} e^{iu_2} = e^{u_1} (\cos u_2 + i \sin u_2)$
Kompleksne kotne funkcije	$\cos \varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2}$ $\sin \varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$ $\cos \hat{u} = \frac{e^{i\hat{u}} + e^{-i\hat{u}}}{2}$ $\sin \hat{u} = \frac{e^{i\hat{u}} - e^{-i\hat{u}}}{2i}$

Kompleksne funkcije
skalarja

$$\hat{u}(t) = u_1(t) + iu_2(t)$$

Diferencial in integral

$$\frac{d\hat{u}}{dt} = \frac{du_1}{dt} + i \frac{du_2}{dt}$$

$$\int \hat{u} dt = \int u_1 dt + i \int u_2 dt$$

Harmonične vrste

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt, \quad n = 1, 2, 3 \dots$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt, \quad n = 1, 2, 3 \dots$$

$$\frac{1}{T} \int_0^T f(t)^2 dt = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

Kompleksne
harmonične vrste

$$f(t) = \operatorname{Re} \sum_{n=-\infty}^{\infty} \hat{A}_n e^{in\omega t}$$

$$\hat{A}_n = \frac{1}{T} \int_0^T f(t) e^{-in\omega t} dt, \quad n = 0, \pm 1, \pm 2 \dots$$

$$\frac{1}{T} \int_0^T f(t)^2 dt = \sum_{n=-\infty}^{\infty} |\hat{A}_n|^2$$

Harmonični integrali

$$f(t) = \operatorname{Re} \int_{-\infty}^{\infty} \hat{A}(\omega) e^{i\omega t} d\omega$$

$$\hat{A}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)^2 dt = \int_{-\infty}^{\infty} |\hat{A}(\omega)|^2 d\omega$$

Vektorji in matrike

Vektor premika

$$\mathbf{r} = (x, y, z)$$

$$r^2 = x^2 + y^2 + z^2$$

Zasuk koordinatnega sistema okoli osi z

$$\begin{aligned}x' &= +x \cos \varphi + y \sin \varphi \\y' &= -x \sin \varphi + y \cos \varphi \\z' &= z\end{aligned}$$

$$x'^2 + y'^2 + z'^2 = x^2 + y^2 + z^2$$

Razteg in vsota

$$\begin{aligned}\lambda \mathbf{u} &= (\lambda u_1, \lambda u_2, \lambda u_3) \\ \mathbf{u} + \mathbf{v} &= (u_1 + v_1, u_2 + v_2, u_3 + v_3)\end{aligned}$$

Enotni vektorji

$$\begin{aligned}\mathbf{e}_1 &= (1, 0, 0) \\ \mathbf{e}_2 &= (0, 1, 0) \\ \mathbf{e}_3 &= (0, 0, 1) \\ \mathbf{u} &= u_1 \mathbf{e}_1 + u_2 \mathbf{e}_2 + u_3 \mathbf{e}_3 = \sum u_i \mathbf{e}_i\end{aligned}$$

Skalarni produkt

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= uv \cos \varphi \\ \mathbf{u} \cdot \mathbf{v} &= u_1 v_1 + u_2 v_2 + u_3 v_3 \\ \mathbf{u} \cdot \mathbf{u} &= u_1^2 + u_2^2 + u_3^2 = u^2\end{aligned}$$

Vektorski produkt

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= uv \sin \varphi \cdot \mathbf{n} \\ \mathbf{u} \times \mathbf{v} &= (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)\end{aligned}$$

Sorazmernost vektorjev

$$\begin{aligned}u_1 &= A_{11}x_1 + A_{12}x_2 + A_{13}x_3 \\ u_2 &= A_{21}x_1 + A_{22}x_2 + A_{23}x_3 \\ u_3 &= A_{31}x_1 + A_{32}x_2 + A_{33}x_3\end{aligned}$$

$$u_i = \sum_j A_{ij}x_j$$

Sorazmernostna matrika

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\mathbf{u} = \mathbf{A} \cdot \mathbf{x}$$

Posebne matrike

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$\mathbf{R}_3 = \begin{vmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Računske operacije

$$\begin{aligned} \lambda \mathbf{A} = \mathbf{B} &\iff B_{ij} = \lambda A_{ij} \\ \mathbf{A} + \mathbf{B} = \mathbf{C} &\iff C_{ij} = A_{ij} + B_{ij} \\ \mathbf{A} \cdot \mathbf{B} = \mathbf{C} &\iff C_{ij} = \sum_k A_{ik} B_{kj} \end{aligned}$$

Inverzna matrika

$$\begin{aligned} \mathbf{A} \cdot \mathbf{x} = \mathbf{u} &\iff \mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{u} \\ \mathbf{A}^{-1} \cdot \mathbf{A} &= \mathbf{I} \end{aligned}$$

$$\begin{aligned} [I_{ij}]^{-1} &= \text{diag}(1, 1, 1) \\ [D_{ij}]^{-1} &= \text{diag}(1/\lambda_1, 1/\lambda_2, 1/\lambda_3) \\ [R_{ij}]^{-1} &= [R_{ji}] \end{aligned}$$

Nje izračun

$$[\mathbf{A} | \mathbf{I}] \rightarrow [\mathbf{I} | \mathbf{A}^{-1}]$$

Večkratne funkcije

Vektorska funkcija
skalarja

$$\mathbf{u}(t) = [u_1(t), u_2(t), u_3(t)]$$

Diferencial in integral

$$\begin{aligned} \mathbf{u}' &= \lim_{dt \rightarrow 0} \frac{\mathbf{u}(t+dt) - \mathbf{u}(t)}{dt} \\ \mathbf{u}' &= (u_1', u_2', u_3') \\ d\mathbf{u} &= \mathbf{u}' \cdot dt \\ d\mathbf{u} &= (du_1, du_2, du_3) \end{aligned}$$

$$\mathbf{u}(t) = \mathbf{u}(0) + \frac{\mathbf{u}'(0)}{1!} t + \frac{\mathbf{u}''(0)}{2!} t^2 + \dots$$

$$\mathbf{u}(t_0 + h) = \mathbf{u}(t_0) + \frac{\mathbf{u}'(t_0)}{1!} h + \frac{\mathbf{u}''(t_0)}{2!} h^2 + \dots$$

$$\mathbf{u} = \int \mathbf{u}' dt = (\int u_1' dt, \int u_2' dt, \int u_3' dt)$$

Skalarna funkcija več
spremenljivk

$$u = u(x, y)$$

Parcialni odvodi

$$u_x = \lim_{dx \rightarrow 0} \frac{u(x+dx, y) - u(x, y)}{dx}$$

Totalni diferencial

$$\begin{aligned} du &= (du)_x + (du)_y = u_x dx + u_y dy \\ \frac{(du)_x}{dx} &= \frac{\partial u}{\partial x} = u_x \end{aligned}$$

Verižna odvajanja

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt}$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t}$$

Implicitno odvajanje

$$F(x,y,u) = 0 \implies F_x + F_u \frac{\partial u}{\partial x} = 0$$

Razvoj v potenčno vrsto

$$u(x,y) = u(0,0) + xu_x + yu_y + \frac{1}{2}(x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}) + \dots$$

$$u(a+h,b+k) = u(a,b) + \frac{1}{1!} \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right) u + \frac{1}{2!} \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^2 u + \dots$$

Ekstremi

$$u = \max \iff u_{xx} < 0 \text{ in } u_{xx}u_{yy} - u_{xy}^2 > 0$$
$$u = \min \iff u_{xx} > 0 \text{ in } u_{xx}u_{yy} - u_{xy}^2 > 0$$

Vezani ekstremi

$$\varphi(x,y) = 0$$
$$u_x + \lambda \varphi_x = 0$$
$$u_y + \lambda \varphi_y = 0$$

Ploščinski integrali

$$U = \int u \, dS$$
$$U = \int \int u \, dx \, dy = \int_c^d dy \int_a^b u \, dx = \int_a^b dx \int_c^d u \, dy$$
$$U = \int \int u \rho \, d\rho \, d\varphi$$

Prostorninski integrali

$$U = \int u \, dV.$$
$$U = \int \int \int u \, dx \, dy \, dz$$
$$U = \int \int \int u \rho \, d\rho \, d\varphi \, dz$$
$$U = \int \int \int u r^2 \sin \theta \, dr \, d\varphi \, d\theta$$

Krivulje in ploskve

Razdalja med točkama

$$s^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

Premica

$$y = kx$$

$$x = At$$

$$y = Bt$$

Krožnica

$$x^2 + y^2 = r^2$$

	$x = r \cos t$ $y = r \sin t$
Elipsa	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $x = a \cos t$ $y = b \sin t$
Hiperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $x = a / \cos t$ $y = b \tan t$
Parabola	$2py = x^2$ $x = At$ $y = Bt^2$
Vektorski opis krivulje	$\mathbf{r}(t) = (x(t), y(t))$ $\frac{dy}{dx} = \frac{y'}{x'}$ $\frac{d^2y}{dx^2} = \frac{x'y'' - y'x''}{x'^3}$
Dolžina krivulje	$ds = d\mathbf{r} = \mathbf{r}' dt = \sqrt{x'^2 + y'^2} dt$ $s = \int \sqrt{x'^2 + y'^2} dt$
Tangenta	$\boldsymbol{\tau} = \frac{d\mathbf{r}}{ds}$ $\boldsymbol{\tau} = \frac{(x', y')}{\sqrt{x'^2 + y'^2}}$
Ukrivljenost	$K = \left \frac{d\boldsymbol{\tau}}{ds} \right $ $K = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$ $K = \frac{1}{R}$
Ravnina	$z = k_1x + k_2y$
Krogla	$x^2 + y^2 = r^2$

$$\begin{aligned}x &= R \sin \theta \cos \varphi \\y &= R \sin \theta \sin \varphi \\z &= R \cos \theta\end{aligned}$$

Vektorski opis ploskve $\mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v))$

Dolžina krivulje na ploskvi $ds^2 = \mathbf{r}_u^2 du^2 + 2\mathbf{r}_u \mathbf{r}_v dudv + \mathbf{r}_v^2 dv^2 = g_{11} du^2 + 2g_{12} dudv + g_{22} dv^2$

$$\begin{aligned}g_{11} &= x_u^2 + y_u^2 + z_u^2 \\g_{12} &= x_u x_v + y_u y_v + z_u z_v \\g_{22} &= x_v^2 + y_v^2 + z_v^2\end{aligned}$$

$$s = \int \sqrt{(g_{11}u'^2 + 2g_{12}u'v' + g_{22}v'^2)} dt$$

Ploščina ploskve $dS = d_u s d_v s \sin \alpha = \sqrt{(g_{11}g_{22} - g_{12}^2)} dudv$
 $S = \int \sqrt{(g_{11}g_{22} - g_{12}^2)} dudv$
 $S = \int \sqrt{(1 + z_x^2 + z_y^2)} dx dy$

Normala $\mathbf{n} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{\sqrt{(g_{11}g_{22} - g_{12}^2)}}$
 $\mathbf{n} = \frac{(-z_x, -z_y, 1)}{\sqrt{(1 + z_x^2 + z_y^2)}}$

Lok na krogli $\cos \alpha = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\varphi_2 - \varphi_1)$

Hipotenuzni izrek $\cos d = \cos a \cos h$

Kosinusni izrek $\cos a = \cos b \cos c + \sin b \sin c \cos A$

Sinusni izrek $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$

Prostorska polja

Skalarno in vektorsko polje $U = U(x, y, z)$
 $\mathbf{v} = (v_x(x, y, z), v_y(x, y, z), v_z(x, y, z))$

Gradient polja $\text{grad } U = \mathbf{n} \cdot \frac{dU}{ds}$
 $\text{grad } U = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) U = \nabla U$
 $dU = \nabla U \cdot d\mathbf{r} = (d\mathbf{r} \cdot \nabla) U$
 $U_2 - U_1 = \int \nabla U ds$

Pretok in divergenca $\Phi = \int \mathbf{v} \cdot \mathbf{n} dS$

$$\int \mathbf{v} \cdot \mathbf{n} dS = \int v_x dy dz + \int v_y dz dx + \int v_z dx dy$$

$$\operatorname{div} \mathbf{v} = \lim_{V \rightarrow 0} \frac{1}{V} \oint \mathbf{v} \cdot \mathbf{n} dS$$

$$\operatorname{div} \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \nabla \cdot \mathbf{v}$$

$$\oint \mathbf{v} \cdot \mathbf{n} dS = \int \nabla \cdot \mathbf{v} dV$$

Cirkulacija in rotor

$$\Gamma = \oint \mathbf{v} ds$$

$$\oint \mathbf{v} ds = \oint v_x dx + \oint v_y dy + \oint v_z dz$$

$$\operatorname{rot} \mathbf{v} = \mathbf{n} \cdot \lim_{S \rightarrow 0} \frac{1}{S} \oint \mathbf{v} ds$$

$$\operatorname{rot} \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}, \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) = \nabla \times \mathbf{v}$$

$$\oint \mathbf{v} ds = \int (\nabla \times \mathbf{v}) \cdot \mathbf{n} dS$$

Operacije drugega reda

$$\nabla \cdot (\nabla U) = \nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$\nabla \times (\nabla U) = 0$$

$$\nabla \cdot (\nabla \times \mathbf{v}) = 0$$

$$\nabla \times (\nabla \times \mathbf{v}) = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$$

Statistika

Število permutacij

$$P_n = n!$$

Število variacij

$$V_n^r = \frac{n!}{(n-r)!}$$

Število kombinacij

$$C_n^r = \frac{n!}{r!(n-r)!}$$

Verjetnost izidov

$$P_k = \lim_{N \rightarrow \infty} \frac{N_k}{N}$$

$$\sum P_k = 1$$

Gostota verjetnosti

$$\frac{dP}{dx} = \lim_{N \rightarrow \infty} \frac{dN(x \pm dx/2)}{N} = p(x)$$

$$\int dP = \int p(x) dx = 1$$

Verjetnost izida A ali B

$$P(A \cup B) = P(A) + P(B)$$

Verjetnost izida A in B	$P(A \cap B) = P(A) \cdot P(B)$
Binomska porazdelitev	$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} = B_{N,p}(n)$
Normalna porazdelitev	$\frac{dP}{dx} = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2\sigma^2} = G_{\mu,\sigma}(x)$
Prehod na novo spremenljivko	$\frac{dP}{dz} = \frac{dP}{dx} \frac{dx}{dz}$
Povprečje	$\langle x \rangle = \int x p(x) dx$
Varianca	$\sigma_x^2 = \int (x - \langle x \rangle)^2 p(x) dx$ $\sigma_x^2 = \int x^2 p(x) dx - (\int x p(x) dx)^2 = \langle x^2 \rangle - \langle x \rangle^2$
Porazdelitev po več spremenljivkah	$\frac{d^2P}{dx dy} = \lim_{N \rightarrow \infty} \frac{dN(x \pm dx/2, y \pm dy/2)}{N} = p(x, y)$
Kumulativna verjetnost	$\frac{dP}{dx} = \int p(x, y) dy = u(x)$
Neodvisni spremenljivki	$p(x, y) = u(x) v(y)$
Povprečje in varianca	$\langle x \rangle = \int x p(x, y) dx dy$ $\sigma_x^2 = \int (x - \langle x \rangle)^2 p(x, y) dx dy$
Kovarianca	$\sigma_{xy} = \int (x - \langle x \rangle)(y - \langle y \rangle) p(x, y) dx dy$
Korelacijski koeficient	$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$
Povprečje povprečij	$\langle \bar{x} \rangle = \langle x \rangle$
Varianca povprečij	$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N}$
Ocena natančnosti meritev	$\langle x \rangle \approx \bar{x} \pm \frac{s_x}{\sqrt{N}}$
Širjenje napak	$u = cx \implies du = c dx$ $u = x \pm y \implies du = dx + dy$ $u = xy \implies \frac{du}{ u } = \frac{dx}{ x } + \frac{dy}{ y }$

$$u = \frac{x}{y} \Rightarrow \frac{du}{|u|} = \frac{dx}{|x|} + \frac{dy}{|y|}$$

$$u = x^n \Rightarrow \frac{du}{|u|} = |n| \frac{dx}{|x|}$$

$$u = u(x) \Rightarrow du = |u'| dx$$

$$u = u(x, y) \Rightarrow du^2 = (u_x dx)^2 + (u_y dy)^2$$

Dinamika

Lega	$\mathbf{r} = (x, y, z)$
Premik	$d\mathbf{r} = d\mathbf{s} = (dx, dy, dz)$
Hitrost	$\mathbf{v} = \frac{d\mathbf{s}}{dt} = (v_x, v_y, v_z)$
Pospešek	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (a_x, a_y, a_z)$
Kotni zasuk	$d\boldsymbol{\varphi} = \mathbf{e}_r \times \frac{d\mathbf{s}}{r}$
Kotna hitrost	$\boldsymbol{\omega} = \frac{d\boldsymbol{\varphi}}{dt}$
Kotni pospešek	$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt}$
Dolžinske in kotne povezave	$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$ $\mathbf{a}_r = -\omega^2 \mathbf{r}$ $\mathbf{a}^2 = \mathbf{a}_t^2 + \mathbf{a}_r^2$
Gibalni zakon za delec	$\mathbf{F} = m \frac{d\mathbf{v}}{dt} = m \frac{d^2\mathbf{r}}{dt^2}$
Izrek o kinetični energiji	$\int \mathbf{F} \cdot d\mathbf{s} = \frac{m\mathbf{v}_2^2}{2} - \frac{m\mathbf{v}_1^2}{2}$
Gibalni zakon za sistem delcev	$\mathbf{F} = m \frac{d^2\mathbf{r}^*}{dt^2}$
Lega težišča	$\mathbf{r}^* = \frac{1}{m} \sum m_i \mathbf{r}_i$

Hitrost težišča	$\mathbf{v}^* = \frac{d\mathbf{r}^*}{dt}$
Gibalna količina sistema	$\mathbf{G} = \sum m_i \mathbf{v}_i = m\mathbf{v}^*$
Izrek o gibalni količini	$\int \mathbf{F} dt = \Delta \mathbf{G}$
Sila curka	$F = \dot{\Phi}_m v$
Raketna enačba	$v - v_0 = u \ln \frac{m_0}{m}$
Vrtilna količina	$\mathbf{L} = \sum \mathbf{r}_i \times m_i \mathbf{v}_i$
Izrek o vrtilni količini	$\int \mathbf{M} dt = \Delta \mathbf{L}$
	$\mathbf{L} = \mathbf{r}^* \times m\mathbf{v}^* + \mathbf{L}^*$
Kinetična energija	$K = \sum \frac{1}{2} m_i \mathbf{v}_i^2$
Izrek o kinetični energiji	$A_{\text{ext}} + A_{\text{int}} = \Delta K$
	$K = \frac{1}{2} m\mathbf{v}^{*2} + K^*$
Vrtenje togega sistema	$L = J\omega$
	$J = \sum m_i r_i^2$
	$M = \frac{dJ\omega}{dt} = J \frac{d\omega}{dt}$
	$K = \frac{1}{2} J\omega^2$
	$J = J^* + mr^{*2}$
	$L = L^* + mr^{*2}\omega$
	$K = K^* + \frac{1}{2} mr^{*2}\omega^2$
Težno nihanje telesa	$\omega_0^2 = \frac{mgl}{ml^2 + J^*}$

Sučno nihanje telesa	$\omega_0^2 = \frac{D}{J^*}$
Prosto nihanje	$u'' + \omega_0^2 u = 0$ $u = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$
Vzbujeno nihanje	$u'' + \omega_0^2 u = A \cos(\omega t + \delta)$ $u = u_0 \cos(\omega t + \delta)$ $u_0 = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)}}$
Vzbujeno nihanje z dušenjem	$u'' + \gamma u' + \omega_0^2 u = A \cos(\omega t + \delta)$ $u = u_0 \cos(\omega t + \delta + \theta)$ $u_0 = \frac{A}{\sqrt{[(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2]}}$ $\tan \theta = \frac{-\gamma \omega}{\omega_0^2 - \omega^2}$
Dušeno nihanje	$u'' + \gamma u' + \omega_0^2 u = 0$ $u = u_0 e^{-\gamma t/2} \cos(\omega t + \delta)$ $\omega = \sqrt{(\omega_0^2 - \gamma^2)}, \gamma < \omega_0$
Gravitacijsko polje	$\mathbf{F}_g = m\mathbf{g}$
Polje okrog masne točke	$\mathbf{g}_P = -\kappa \frac{m_Q}{r_{PQ}^2} \mathbf{n}_{PQ}, \quad \kappa = 6,7 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$
Superpozicija polj	$\mathbf{g}_P = - \sum_Q \kappa \frac{m_Q}{r_{PQ}^2} \mathbf{n}_{PQ}$
Gravitacijska energija	$W - W_0 = - \int m\mathbf{g} \cdot d\mathbf{s}$
Gravitacijski potencial	$\phi - \phi_0 = - \int \mathbf{g} \cdot d\mathbf{s}$
Potencial masne točke	$\phi_P = -\kappa \frac{m_Q}{r_{QP}}$

Superpozicija
potencialov

$$\phi_P = - \sum_Q \kappa \frac{m_Q}{r_{QP}}$$

Gibanje planeta

$$L = mr^2\dot{\varphi}$$

$$E = \frac{m}{2} (\dot{r}^2 + r^2\dot{\varphi}^2) - \frac{\alpha}{r}$$

$$r = \frac{p}{1 + \varepsilon \cos \varphi}$$

$$p = \frac{L^2}{m\alpha}$$

$$\varepsilon^2 = 1 + \frac{2EL^2}{m\alpha^2}$$

$$\frac{T^2}{a^3} = \frac{4\pi^2}{\kappa M}$$

Relativnost

Stalnost svetl. hitrosti

$$c' = c$$

Relativistični faktor

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Transformacija
koordinat

$$t' = \gamma(t - ux/c^2)$$

$$x' = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

Transformacija
intervalov

$$\Delta t = t_B - t_A$$

$$\Delta x = x_B - x_A$$

$$\Delta t' = \gamma(\Delta t - u\Delta x/c^2)$$

$$\Delta x' = \gamma(\Delta x - u\Delta t)$$

Podaljšanje časa

$$\Delta t = \gamma\Delta t_0$$

Skrajšanje dolžin

$$\Delta l = \Delta l_0 / \gamma$$

Transformacija hitrosti

$$v = \frac{x_B - x_A}{t_B - t_A}$$

$$v_x' = \frac{v_x - u}{1 - v_x u / c^2}$$

	$v_y' = \frac{1}{\gamma} \frac{v_y}{1 - v_x u/c^2}$ $v_z' = \frac{1}{\gamma} \frac{v_z}{1 - v_x u/c^2}$
Frekvenčni zamik svetlobe	$\frac{\nu'}{\nu} = \frac{\sqrt{1 - u/c}}{\sqrt{1 + u/c}}$
Sprememba valovne dolžine	$\frac{\lambda'}{\lambda} = \frac{\sqrt{1 + u/c}}{\sqrt{1 - u/c}}$
Približek za majhne hitrosti	$\frac{\Delta\lambda}{\lambda} = \frac{u}{c}$
Gibalna količina	$\mathbf{G} = m\gamma(v)\mathbf{v}$
Gibalni zakon	$\frac{d\mathbf{G}}{dt} = \mathbf{F}$
Polna energija	$E = \gamma(v) mc^2$
Kinetična in masna energija	$E = K + mc^2$ $K = (\gamma(v) - 1)mc^2$
Energijska invarianta	$E^2 - (cG)^2 = (mc^2)^2$
Transformacija G in E	$G_x' = \gamma(G_x - uE/c^2)$ $E' = \gamma(E - uG_x)$ $\Delta G_x' = \gamma(\Delta G_x - u\Delta E/c^2)$ $\Delta E' = \gamma(\Delta E - u\Delta G_x)$
Gibalna količina svetlobe	$G = \frac{E}{c}$
Gostota gibalne količine	$g = \frac{G}{V}$ $g = \frac{j}{c^2}$
Svetlobni tlak	$p = gc$ $p = (1 + R) gc$

Termodinamika

Številna gostota molekul	$n = \frac{N}{V}$
Plinski zakon	$p = nkT, \quad k = \frac{R^*}{N_A}$
Kinetični model tlaka	$p = \frac{1}{3} nm_1 \langle v^2 \rangle$
Kinetični model temperature	$\langle \frac{1}{2} m_1 v^2 \rangle = \frac{3}{2} kT$
Porazdelitev molekul po višini	$\frac{dP}{dz} = A e^{-m_1 g z / kT}, \quad A = \frac{m_1 g}{kT}$
Porazdelitev molekul po hitrosti	$\frac{dP}{dv_x} = A e^{-m_1 v_x^2 / 2kT}$ $\frac{dP}{dv} = A^3 4\pi v^2 e^{-m_1 v^2 / 2kT}, \quad A = \left(\frac{m_1}{2\pi kT} \right)^{1/2}$
Porazdelitev molekul po energiji	$P_i = \frac{1}{Z} e^{-E_i / kT}, \quad Z = \sum e^{-E_i / kT}$ $dP = \frac{1}{Z} e^{-E / kT} dE, \quad Z = \int e^{-E / kT} dE$
Porazdelitev po faznem prostoru	$dP = \frac{1}{Z} e^{-E(r,v) / kT} d^3r d^3v, \quad Z = \int e^{-E(r,v) / kT} d^3r d^3v$
Notranja energija plina	$U = N \frac{f}{2} kT$
Toplotna kapaciteta plina	$C_V = \frac{U}{T} = N \frac{f}{2} k$
Specifične toplote plina	$c_V = \frac{C_V}{Nm_1} = \frac{f}{2} \frac{k}{m_1} = \frac{f}{2} \frac{R^*}{M}$ $c_p = c_V + \frac{R^*}{M} = \left(\frac{f}{2} + 1 \right) \frac{R^*}{M}$ $\kappa = \frac{c_p}{c_V} = 1 + \frac{2}{f}$

Kristalna snov	$U = 3NkT$ $c_V = 3 \frac{R^*}{M}$
Izhlapenjanje tekočine	$n_v = \frac{1}{V_0} e^{-W/kT}$
Ionizacija plina	$\frac{n_e n_i}{n_a} = \frac{1}{V_0} e^{-W/kT}$
Prosta pot molekul	$\langle l \rangle = \frac{1}{\pi(2r)^2 n}$
Difuzivnost	$j_m = -D \frac{d\rho}{dx}, \quad D = \frac{1}{3} \langle v \rangle \langle l \rangle$
Toplotna prevodnost	$j_Q = -\lambda \frac{dT}{dx}, \quad \lambda = \frac{1}{6} \langle v \rangle \langle l \rangle n f k$
Viskoznost	$\eta = \frac{1}{3} \langle v \rangle \langle l \rangle n m_1$
Transportni koeficienti	$\eta = \rho D = \lambda / c_V$
Difuzijska enačba	$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2}$
Difuzija tanke plasti	$\rho(x, t) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp \frac{-x^2}{2\sigma_x^2}, \quad \sigma_x^2 = 2Dt$
Difuzijska enačba toplote	$\frac{\partial T}{\partial t} = D^* \frac{\partial^2 T}{\partial x^2}, \quad D^* = \frac{\lambda}{c_p \rho}$
Pijančeva hoja	$\langle r^2 \rangle = 6Dt$
Prsilna hitrost	$v_{\text{drift}} = \frac{\langle \tau \rangle}{m_1} F = \mu F$
Difuzivnost in gibljivost	$D = \mu kT$
Difuzija kroglic v tekočini	$D = \frac{kT}{6\pi\eta r}$

Statična E & M polja

Električna sila in polje

$$\mathbf{F}_e = e\mathbf{E}$$

Polje točkastega naboja

$$\mathbf{E}_P = \kappa_e \frac{e_Q}{r_{QP}^2} \mathbf{n}_{QP}$$

Superpozicija polj

$$\mathbf{E}_P = \kappa_e \sum_Q \frac{e_Q}{r_{QP}^2} \mathbf{n}_{QP}$$

$$\mathbf{E}_P = \kappa_e \int \frac{\rho_Q dV_Q}{r_{QP}^2} \mathbf{n}_{QP}$$

Pretok in divergenca polja

$$\oint \mathbf{E} \cdot d\mathbf{S} = 4\pi \kappa_e e = \frac{e}{\epsilon_0}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Cirkulacija in rotor polja

$$\oint \mathbf{E} \cdot d\mathbf{s} = 0$$

$$\nabla \times \mathbf{E} = 0$$

Električni potencial

$$\mathbf{E} = -\nabla U$$

$$U_B - U_A = - \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Potencial točkastega naboja

$$U_P = \frac{1}{4\pi\epsilon_0} \frac{e_Q}{r_{QP}}$$

Superpozicija potencialov

$$U_P = \frac{1}{4\pi\epsilon_0} \sum_Q \frac{e_Q}{r_{QP}}$$

$$U_P = \frac{1}{4\pi\epsilon_0} \int \frac{\rho_Q dV_Q}{r_{QP}}$$

Potencialna enačba

$$\nabla^2 U = - \frac{\rho}{\epsilon_0}$$

Električni dipol

$$\mathbf{p}_e = e\mathbf{d}$$

$$U = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p}_e \cdot \mathbf{e}_r}{r^2}$$

Navor na dipol	$\mathbf{M} = \mathbf{p}_e \times \mathbf{E}$
Sila na dipol	$\mathbf{F} = (\mathbf{p}_e \cdot \nabla) \mathbf{E}$
Energija dipola	$W = -\mathbf{p}_e \cdot \mathbf{E}$
Polarizacija snovi	$\mathbf{P} = \frac{d\mathbf{p}_e}{dV} = n\mathbf{e}\mathbf{d}$
Električna susceptibilnost	$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$ $\varepsilon = 1 + \chi_e$
Magnetna sila in polje	$\mathbf{F}_m = I\mathbf{l} \times \mathbf{B}$
Polje tokov	$\mathbf{B}_P = \kappa_m \oint \frac{I d\mathbf{s}_Q \times \mathbf{n}_{QP}}{r_{QP}^2}$ $\mathbf{B}_P = \kappa_m \oint \frac{\mathbf{j}_Q \times \mathbf{n}_{QP}}{r_{QP}^2} dV_Q$
Pretok in divergenca polja	$\oint \mathbf{B} \cdot d\mathbf{S} = 0$ $\nabla \cdot \mathbf{B} = 0$
Cirkulacija in rotor polja	$\oint \mathbf{B} \cdot d\mathbf{s} = 4\pi\kappa_m I = \mu_0 I$ $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$
Magnetni potencial	$\mathbf{B} = \nabla \times \mathbf{A}$
Potencialna enačba	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{j}$
Potencial tokov	$\mathbf{A}_P = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}_Q dV_Q}{r_{QP}}$
Magnetni dipol	$\mathbf{p}_m = I\mathbf{S}$ $\mathbf{A} = \frac{\mu_0}{4\pi} \frac{\mathbf{p}_m \times \mathbf{e}_r}{r^2}$
Navor na dipol	$\mathbf{M} = \mathbf{p}_m \times \mathbf{B}$
Sila na dipol	$\mathbf{F} = \nabla (\mathbf{p}_m \cdot \mathbf{B}) = (\mathbf{p}_m \cdot \nabla) \mathbf{B}$
Energija dipola	$W = -\mathbf{p}_m \cdot \mathbf{B}$

Magnetizacija snovi

$$\mathbf{M} = \frac{d\mathbf{p}_m}{dV} = nI\mathbf{S}$$

Magnetna
susceptibilnost

$$\mathbf{M} = \frac{\chi_m}{\mu_0} \mathbf{B}$$

$$\frac{1}{\mu} = 1 - \chi_m$$

Relativnost gostote
nabojev in tokov

$$\rho' = \gamma(\rho - u j_x / c^2)$$

$$j'_x = \gamma(j_x - u\rho)$$

$$j'_y = j_y$$

$$j'_z = j_z$$

Relativnost polj

$$E'_y = \gamma(E_y - uB_z)$$

$$B'_z = \gamma(B_z - uE_y / c^2)$$

$$E'_z = \gamma(E_z + uB_y)$$

$$B'_y = \gamma(B_y + uE_z / c^2)$$

$$E'_x = E_x$$

$$B'_x = B_x$$

Elektromagnetni valovi

Osnovne enačbe polja

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Polje brez nabojev in
tokov

$$\nabla^2 \mathbf{E} - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{B} - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

Ravno valovanje $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$
 $\mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$

$$\mathbf{E} = c\mathbf{B} \times \mathbf{n}$$

Stojno valovanje $\mathbf{E} = \mathbf{E}_0(\mathbf{r}) e^{-i\omega t}$

$$\nabla^2 \mathbf{E}_0 = -k^2 \mathbf{E}_0$$

$$k^2 = \omega^2/c^2$$

Energija valovanja

$$\frac{\partial w}{\partial t} = -\nabla \cdot \mathbf{J} - \mathbf{j} \cdot \mathbf{E}$$

$$w = \frac{\epsilon_0}{2} \mathbf{E}^2 + \frac{1}{2\mu_0} \mathbf{B}^2$$

$$\mathbf{J} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

Energija v ravnem valovanju

$$\langle w \rangle = \frac{\epsilon_0}{2} E_0^2 = \frac{1}{2\mu_0} B_0^2$$

$$\langle \mathbf{J} \rangle = c \langle w \rangle$$

Razklopitev osnovnih enačb

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla U - \frac{\partial \mathbf{A}}{\partial t}$$

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{j}$$

$$\nabla^2 U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

Njihova rešitev

$$U_P(t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho_Q(t - r_{QP}/c) dV_Q}{r_{QP}}$$

$$\mathbf{A}_P(t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}_Q(t - r_{QP}/c) dV_Q}{r_{QP}}$$

Dipolno sevanje

$$\langle P \rangle = \frac{p_0^2 \omega^4}{12\pi\epsilon_0 c^3}$$

$$\langle I \rangle = \frac{3 \langle P \rangle}{2 4\pi} \sin^2 \theta$$

Osnovne enačbe v snovi

$$\nabla \cdot \epsilon \mathbf{E} = \frac{\rho_{\text{free}}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \frac{\mathbf{B}}{\mu} = \mu_0 \mathbf{j}_{\text{free}} + \frac{1}{c^2} \frac{\partial \epsilon \mathbf{E}}{\partial t}$$

Mejni pogoji

$$B_{\perp}(1) = B_{\perp}(2)$$

$$\epsilon_1 E_{\perp}(1) = \epsilon_2 E_{\perp}(2)$$

$$E_{\parallel}(1) = E_{\parallel}(2)$$

$$B_{\parallel}(1)/\mu_1 = B_{\parallel}(2)/\mu_2$$

Valovanje v dielektriku

$$v = \frac{c}{\sqrt{\epsilon \mu}}$$

$$n = \sqrt{\epsilon \mu}$$

Odbojni količnik

$$R_{\perp} = \left| \frac{n_1 \cos \alpha - n_2 \cos \alpha_2}{n_1 \cos \alpha + n_2 \cos \alpha_2} \right|^2$$

$$R_{\parallel} = \left| \frac{n_2 \cos \alpha - n_1 \cos \alpha_2}{n_2 \cos \alpha + n_1 \cos \alpha_2} \right|^2$$

$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$$

Uklon za odprtino

$$u_p \propto \int \exp(iks) dS$$

Mrežica tankih rež

$$j(\alpha) = j_0 \left[\frac{\sin(1/2 Nka \sin \alpha)}{N \sin(1/2 ka \sin \alpha)} \right]^2$$

Široka reža

$$j(\alpha) = j_0 \left[\frac{\sin(ka \sin \alpha)}{ka \sin \alpha} \right]^2$$

Elektroni in ioni

Sila na elektron

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Hitrost pospešenega elektrona

$$\frac{1}{2} m v^2 = e U_A$$

Odklon v električnem polju

$$\theta = \frac{e E l}{m v^2}$$

Odklon v magnetnem polju	$\phi = \frac{eBs}{mv}$
Brez odklona v prekržanih poljih	$v = \frac{E}{B}$
Relativistični elektroni	$\theta = \frac{eEl}{\gamma mv^2}$
	$\phi = \frac{eBs}{\gamma mv}$
Nepolarna molekula	$\mathbf{p}_e = \alpha \epsilon_0 \mathbf{E}$
Plin nepolarnih molekul	$\frac{M}{\rho} (\epsilon - 1) = N_A \alpha$
Polarna molekula	$\langle p_e \rangle = \frac{p_0^2}{3kT} E$
Plin polarnih molekul	$\frac{M}{\rho} (\epsilon - 1) = N_A \left(\alpha + \frac{p_0^2 / \epsilon_0}{3kT} \right)$
Nemagnetna molekula	$\mathbf{p}_m = \beta \mathbf{B} / \mu_0$
Plin nemagnetnih molekul	$\frac{M}{\rho} (\mu - 1) = N_A \beta$
Magnetna molekula	$\langle p_m \rangle = \frac{p_0^2}{3kT} B$
Plin magnetnih molekul	$\frac{M}{\rho} (\mu - 1) = N_A \left(\beta + \frac{p_0^2 \mu_0}{3kT} \right)$
Elektroni v prevodniku	$\mathbf{v}_{\text{drift}} = \frac{e \mathbf{E} \tau}{m}$
	$\mathbf{j} = ne \mathbf{v}_{\text{drift}} = \sigma \mathbf{E}$
	$\sigma = \frac{ne^2 \tau}{m}$
Prečna napetost	$U = \frac{1}{ne} \frac{IB}{d}$

Elektronski oscilatorji

$$n - 1 = \frac{e^2/m}{2\varepsilon_0(\omega_0^2 - \omega^2)} \frac{N}{V}$$

$$n - 1 = \frac{N}{V} \sum f_k \frac{e^2/m}{2\varepsilon_0(\omega_k^2 - \omega^2)}$$

Kvantni delci

Fotoelektrični pojav

$$K = h\nu - W, \quad h = 6,63 \cdot 10^{-34} \text{ Js}$$

Energija fotona

$$E = h\nu$$

Gibalna količina
fotona

$$G = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

Valovna dolžina
elektrona

$$\lambda = \frac{h}{G} = \frac{h}{mv}$$

Sipanje alfa na
atomskih jedrih

$$b = \frac{1}{2} \left(\frac{Z_1 Z_2 q^2}{mv^2/2} \right) \cot \frac{\theta}{2}$$

$$\frac{dI/I_0}{d\Omega} = \frac{1}{16} \frac{N}{V} \left(\frac{Z_1 Z_2 q^2}{mv^2/2} \right)^2 \frac{l}{\sin^4 \theta/2}$$

Energijski nivoji
atomov

$$h\nu = E_2 - E_1$$

Kvantizacija krožnic

$$2\pi r_n = n\lambda, \quad n = 1, 2, 3 \dots$$

Kvantizacija vrtilne
količine

$$L = mvr = n\hbar, \quad n = 1, 2, 3 \dots$$

$$\hbar = h/2\pi$$

Krožnice v atomu
vodika

$$r = \frac{\hbar^2}{mq^2} \cdot n^2 = r_B \cdot n^2, \quad n = 1, 2, 3 \dots$$

Obodna hitrost v
atomu vodika

$$\alpha = \frac{v}{c} = \frac{q^2}{\hbar c} = 1/137$$

Energijski nivoji v
atomu vodika

$$E = - \frac{mq^4}{2\hbar^2} \frac{1}{n^2} = E_R \cdot \frac{1}{n^2}$$

Krožnice ali elipse v
atomu vodika

$$E = - \frac{q^2}{2r} = - \frac{q^2}{2a}$$

Velike polosi v atomu
vodika

$$a = r_B \cdot n^2$$

Male polosi v atomu
vodika

$$b = \frac{l}{n} a$$

Vrtilne količine elips v
atomu vodika

$$L = l\hbar, \quad l = 1, 2, 3 \dots n$$

Slojevita zgradba
atomov

$$r_n = r_B \frac{n^2}{Z-S}$$
$$E_n = E_R \frac{(Z-S)^2}{n^2}$$

Notranji elektroni

$$\nu = \frac{3}{4} R_y (Z-1)^2$$

Orbitalni magnetni
moment

$$\mu = \frac{e}{2m} L$$

$$\mu = \frac{e}{2m} \hbar l = \mu_B l, \quad l = 1, 2, 3 \dots n$$

Njegova precesija

$$\Omega = \frac{e}{2m} B$$

Njegova smerna
kvantizacija

$$L_z = m_l \hbar$$
$$\mu_z = m_l \mu_B$$
$$m_l = -l, -l+1 \dots -1, 0, 1 \dots l-1, l$$

Spin in spinski
magnetni moment

$$L = s\hbar, \quad s = 1/2$$
$$L_z = m_s \hbar, \quad m_s = -s, +s$$

$$\mu_z = 2 \cdot \frac{e}{2m} L_z = \pm \hbar$$

Celotni magnetni
moment

$$L = j\hbar$$
$$\mu = j\mu_B$$
$$L_z = m_j \hbar$$
$$\mu_z = m_j \mu_B$$
$$m_j = -j, -j+1 \dots j-1, j$$

Valovna mehanika

Valovna funkcija delca

$$\Psi(\mathbf{r}, t)$$

Verjetnost lege

$$\frac{dP}{dV} = |\Psi|^2$$

	$\int \Psi ^2 dV = 1$
Ravni val	$\Psi(x, t) = Ae^{i(kx - \omega t)} = Ae^{i(Gx - Et)/\hbar}$
Valovni paket	$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int A(k)e^{ikx} dk$
Njegov spekter	$A(k) = \frac{1}{\sqrt{2\pi}} \int \Psi(x)e^{-ikx} dx$
	$\frac{dP}{dk} = A ^2$
	$\int A ^2 dk = 1$
Razmazanost paketa	$\Delta x^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$ $\Delta G^2 = \langle (G - \langle G \rangle)^2 \rangle = \langle G^2 \rangle - \langle G \rangle^2$
	$\Delta x \Delta G \geq \frac{\hbar}{2}$
Kvantni gibalni zakon	$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + W(\mathbf{r})\Psi$
Tok verjetnosti	$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$ $\mathbf{j} = \frac{\hbar}{2mi} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*)$
Stacionarna stanja	$\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-i\omega t}$
Kvantna amplitudna enačba	$-\frac{\hbar^2}{2m} \nabla^2 \psi + [W(\mathbf{r}) - E]\psi = 0$
Čisto stanje	$\Psi(x, t) = \psi_n(x) e^{-iE_n t/\hbar}$
Mešano stanje	$\Psi(x, t) = \sum c_n \psi_n(x) e^{-iE_n t/\hbar}$
	$\int \psi_m^* \psi_n dV = 1, \text{ če } n = m, \text{ sicer } 0$
	$\sum c_n ^2 = 1$
	$P(E_n) = c_n ^2$

Vpad na stopnico

$$R = \frac{k_1 - k_2}{k_1 + k_2}$$
$$T = \frac{2k_1}{k_1 + k_2}$$

Delec v potencialni jami

$$E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{D} \right)^2, \quad n = 1, 2, 3 \dots$$

$$\psi_n \propto \sin \frac{n\pi}{D} x$$

Harmonični oscilator

$$W = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2$$

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2, 3 \dots$$

$$\psi_n \propto H_n \left(\frac{x}{a} \right) \exp \left(-\frac{x^2}{2a^2} \right), \quad a^2 = \frac{\hbar}{m\omega}$$

$$\psi_0 \propto \exp \left(-\frac{x^2}{2a^2} \right)$$

Vodikov atom

$$W = -\frac{q^2}{r}$$

$$E_n = \frac{-mq^4}{2\hbar^2} \frac{1}{n^2}$$

$$\psi_{nlm}(r, \theta, \varphi) \propto R_{nl}(r) P_{lm}(\cos \theta) \Phi_m(\varphi)$$

$$\psi_{100} \propto \exp \left(-\frac{r}{r_B} \right)$$

$$n = 1, 2, 3 \dots$$

$$l = 0, 1, 2 \dots n - 1$$

$$m = 0, \pm 1, \pm 2 \dots \pm l$$

Vrtilna količina

$$-i\hbar(\mathbf{r} \times \nabla)\psi = \mathbf{L}\psi$$

$$L^2 = l(l+1)\hbar^2, \quad l = 0, 1, 2, 3, \dots$$

Projekcija vrtilne količine

$$-i\hbar \frac{\partial \psi}{\partial \varphi} = L_z \psi$$

$$L_z = m\hbar, \quad m = 0, \pm 1, \pm 2, \dots \pm l$$

Večelektronski atom

$$\psi(x_1, y_1, z_1, x_2, y_2, z_2) = \psi(1, 2)$$

$$dV = dx_1 dy_1 dz_1 dx_2 dy_2 dz_2$$

$$\frac{dP}{dV} = |\psi|^2$$

$$-\left[\frac{\hbar^2}{2m} \nabla_1^2 + \frac{\hbar^2}{2m} \nabla_2^2 \right] \psi + W(1, 2) \psi = E \psi$$

Neodvisni elektroni

$$W(1, 2) = W(1) + W(2)$$

$$\psi(1, 2) = u(1)v(2)$$

$$-\frac{\hbar^2}{2m} \nabla_1^2 u + W(1)u = E_1 u$$

$$-\frac{\hbar^2}{2m} \nabla_2^2 v + W(2)v = E_2 v$$

$$E = E_1 + E_2$$

Molekule, kristali, plini

Rotacija molekule

$$E = \frac{\hbar^2}{2J} l(l+1), \quad l = 0, 1, 2, 3 \dots$$

Nihanje dvoatomne
molekule

$$E = \hbar\omega_0 \left(n + \frac{1}{2} \right)$$

Porazdelitev molekul
po energiji

$$P_i = \frac{1}{Z} g_i e^{-E_i/kT}$$

$$Z = \sum g_i e^{-E_i/kT}$$

Nihanje dvoatomnih
molekul

$$P_n = \frac{1}{Z} \exp \left(-\frac{\hbar\omega_0}{kT} n \right)$$

$$Z = \frac{1}{1 - \exp(-\hbar\omega_0/kT)}$$

$$\langle E \rangle = \frac{\hbar\omega_0}{\exp(\hbar\omega_0/kT) - 1} = \hbar\omega_0 \langle n \rangle$$

Vrtenje dvoatomnih
molekul

$$P_l = \frac{1}{Z} (2l+1) \exp -\frac{(\hbar^2/2J) l(l+1)}{kT}$$

$$Z \approx \frac{2J}{\hbar^2} kT$$

Nihanje atomov v kristalu

$$U = 3NkT \frac{\hbar\omega/kT}{e^{\hbar\omega/kT} - 1}$$

$$c_V = 3 \frac{k}{m_1} \left(\frac{\theta}{T}\right)^2 \frac{e^{\theta/T}}{(e^{\theta/T} - 1)^2}$$

$$\theta = \frac{\hbar\omega}{k}$$

Fermionska porazdelitev

$$f_i = \frac{N_i}{g_i} = \frac{1}{e^{(E_i - E_F)/kT} + 1}$$

$$\frac{dn}{dE} = \frac{g(E)}{\exp(E - E_F)/kT + 1} = g(E) f(E)$$

Elektronski plin v kovini

$$g = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \sqrt{E}$$

$$E_F = \frac{h^2}{2m} \left(\frac{3n}{8\pi}\right)^{2/3}$$

$$W = \frac{3}{5} N E_F$$

$$p = \frac{2}{3} \frac{W}{V} = \frac{2}{5} \frac{N E_F}{V}$$

$$\frac{1}{2} m v_F^2 = E_F$$

Pogoj za nedegenerirano plazmo

$$\frac{h}{\sqrt{3mkT}} \ll \frac{1}{n^{1/3}}$$

Pogoja za nerelativistično plazmo

$$T \ll \frac{mc^2}{k}$$

$$n \ll \frac{\sqrt{8}}{3\pi^2} \left(\frac{mc}{\hbar}\right)^3$$

Bozonska porazdelitev

$$f_i = \frac{N_i}{g_i} = \frac{1}{e^{(E_i - E_F)/kT} - 1}$$

$$\frac{dN}{dE} = \frac{g(E)}{\exp(E/kT) - 1} = g(E) f(E)$$

Fotoni v votlini

$$g(E) = \frac{8\pi V}{(hc)^3} E^2$$

$$\frac{dw}{dE} = \frac{8\pi}{(hc)^3} \frac{E^3}{e^{E/kT} - 1}$$

$$\frac{dw}{d\nu} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$\frac{dw}{d\lambda} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$$p = \frac{1}{3} \frac{W}{V}$$

$$p = a T^4$$
$$a = \frac{8\pi^5 k^4}{15c^3 h^3}$$

Toplotno sevanje

$$\frac{dB}{d\lambda} = \frac{2hc^3}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$$\frac{dj^*}{d\lambda} = \pi \frac{dB}{d\lambda}$$

$$j^* = \sigma T^4$$
$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$$

$$a = \frac{4\sigma}{3c}$$

$$\lambda_{\max} = \frac{b}{T}$$

$$b = \frac{hc}{4.97k}$$

Atomska jedra

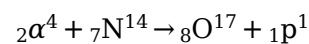
Centralni trk delca 1 v mirujoč delec 2

$$\frac{v_2'}{v_1} = \frac{2m_1}{m_1 + m_2}$$

Trk delca 1 z jedrom 2

$$\frac{m_2}{m_1} = \frac{\sin \theta_1}{\sin(\theta_1 + \theta_2)}$$

Transmutacija dušika



Odkritje nevtrona	${}_2\alpha^4 + {}_4\text{Be}^9 \rightarrow {}_6\text{C}^{12} + {}_0\text{n}^1$
	$m_n = 1,009 u$
Vezavna energija jedra	$E_{\text{bind}} = [Zm_p + (A - Z)m_n - m]c^2$
Vezavna energija nukleona	$B = \frac{E_{\text{bind}}}{A}$
Razpad alfa	${}_Z\text{X}^A \rightarrow {}_{Z-2}\text{Y}^{A-4} + {}_2\alpha^4$
Razpad beta	${}_Z\text{X}^A \rightarrow {}_{Z+1}\text{Y}^A + e^- + \nu$
Razpad gama	${}_Z\text{X}^A \rightarrow {}_Z\text{X}^A + \gamma$
Preostala jedra	$N = N_0 e^{-\lambda t}$
Razpadni čas	$\tau = \frac{1}{\lambda}$
Razpolovni čas	$T_{1/2} = \tau \ln 2$
Aktivnost vira	$A = -\frac{dN}{dt} = A_0 e^{-\lambda t}$
Ravnovesje v verigi razpadov	$\frac{N_A}{T_A} = \frac{N_B}{T_B} = \frac{N_C}{T_C}$
Absorpcijski radij jedra za nevtrone	$r = r_0 A^{1/3}, \quad r_0 = 1,2 \text{ fm}$
Cepitev težkih jeder	$n + \text{U}^{235} \rightarrow \text{X} + \text{Y} + \sim 2,5 n$
Zlivanje lahkih jeder	$\text{H}^2 + \text{H}^3 \rightarrow \text{He}^4 + n^1$

Zvezde in vesolje

Zvezde na glavni veji	$P \propto T_E^8$
	$R \propto T_E^2$
	$P \propto M^4$
	$T_E \propto M^{1/2}$
	$R \propto M$

	$\langle \rho \rangle \propto \frac{1}{M^2}$
Masa sredice oblaka	$M_r = \int_0^r \rho 4\pi r^2 dr$
Gravitacijska energija oblaka	$E_G = - \int \frac{\kappa M_r}{r} dm \sim - \frac{\kappa M^2}{R}$
Toplotna energija oblaka	$E_T = \frac{3}{2} \frac{M}{\bar{m}} kT$
Sesedna masa oblaka	$M_J = \frac{3kT}{2\kappa\bar{m}} R$
Hidrostatsično ravnovesje	$\frac{dp}{dr} = - \frac{\kappa M_r \rho}{r^2}$
Tlak v središču	$p_c \sim \frac{\kappa M^2}{R^4}$
Notranja temperatura zvezde	$T_c \propto \frac{M}{R}$
Masni in fotonski tlak	$\frac{p_{\text{rad}}}{p_{\text{gas}}} \propto M^2$
Vezavna energija zvezde	$E_{\text{bind}} = -(E_T + E_G)$
Za masni plin	$E_T = - \frac{E_G}{2}$ $E_{\text{bind}} = E_T$
Za fotonski plin	$E_T = -E_G$ $E_{\text{bind}} = 0$
Zlivanje protonov v helij	$4p \rightarrow \text{He}^4 + 2e^+ + 2\nu$ ${}^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H} + e^+ + \nu + 0.4 \text{ MeV}$ ${}^1\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma + 5.5 \text{ MeV}$ ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2({}^1\text{H}) + 12.9 \text{ MeV}$
Življenjski čas	$t \propto \frac{1}{M^3}$

Proizvodnja energije	$\frac{dP}{dr} = \epsilon \rho 4\pi r^2$
Difuzijski prenos	$\frac{dT}{dr} = \frac{P}{4\pi r^2 \lambda}$
Konvektivni prenos	$\frac{dT}{dr} = \frac{\gamma - 1}{\gamma} \frac{T}{p} \frac{dp}{dr}$
Radij bele pritlikavke	$R \propto \frac{1}{M^{1/3}}$
Sesedna masa bele pritlikavke	$M \sim \left(\frac{N}{M}\right)^2 \left(\frac{\hbar c}{\kappa}\right)^{3/2}$
Nihanje svetilnosti kefeid	$M = -a \lg(P/\text{dan}) - b$ $a = 2.4$ $b = 1.7$
Beg galaksij	$v = H_0 r$
Trajanje od velikega poka	$t_0 \sim \frac{1}{H_0}$
Skalirni faktor vesolja	$\mathbf{r} = a(t)\mathbf{R}$
Skalirna enačba	$\left(\frac{a'}{a}\right)^2 = \frac{8\pi\kappa}{3}\rho - \frac{\kappa c^2}{a^2}$ $H = \frac{a'}{a}$
Gostotna enačba	$\rho' + 3\frac{a'}{a}\left(\rho + \frac{p}{c^2}\right) = 0$
Raztegovanje svetlobe	$\lambda \propto a$
Ravno masno dominirano vesolje	$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$ $\rho(t) = \frac{\rho_0}{a^3}$
Ravno sevalno dominirano vesolje	$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$

$$\rho(t) = \frac{\rho_0}{a^4}$$

Kritična gostota
vesolja

$$\rho_c = \frac{3H^2}{8\pi K}$$

Ohlajanje prasevanja

$$T \propto \frac{1}{a}$$